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A Semi-Markov Framework for Occupational Mobility and Destination Probability Modelling

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ABSTRACT
This paper presents a semi-Markov process framework for analyzing occupational mobility within hierarchical organizations. Unlike conventional Markov models that assume memoryless transitions, the semi-Markov approach incorporates random holding times, allowing more realistic modeling of promotion delays across grades. The framework distinguishes between open and closed grades, considers multiple-promotion pathways and derives the interval transition probabilities $\phi_{ij}(t)$ and destination probabilities $\gamma_{ijq}(l/t)$, which measure the likelihood that an employee starting at grade i moves through grade j and ultimately reaches grade q within time t. A simulation study using exponential waiting times demonstrates how mobility patterns vary with entry grade, the number of required promotions and the structure of the transition matrix. Results show that higher initial grades improve chances of reaching upper positions, while paths requiring several promotions have lower probabilities.

KEYWORDS
Semi-Markov process, destination probability, exponential distribution, simulation.

1. Introduction

In recent years, the globalization of markets and the rapid advancement of technology have made the study of occupational mobility increasingly important across industries. There is now a growing trend of employees changing jobs frequently in search of better opportunities. In many private organizations, very few employees remain in the same company or in the same occupational category for a long period of time. Since human behaviour is inherently unpredictable, it becomes necessary to describe such mobility patterns in probabilistic terms.

Occupational mobility is better captured by semi-Markov processes than by traditional Markov models. Classical Markov frameworks do not specify the exact moments when transitions occur, whereas in reality individuals shift from one occupational category to another at irregular and uncertain intervals. A realistic stochastic model must therefore incorporate the variability in the timing of these transitions. In a semi-Markov process, the sequence of states evolves according to the transition probabilities of an

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underlying Markov chain, but the duration spent in each state is governed by a positive random variable. This duration depends not only on the current job category but also on the subsequent one. Hence, semi-Markov processes provide a suitable structure for describing occupational mobility, where the states represent different job groups.

A considerable amount of research has examined occupational mobility using semi-Markov approaches. Foundational work was carried out by Ginsberg (1971, 1972a, 1972b, 1973, 1978a, 1978b) [7, 8, 9, 11, 12]. Ugwuowo and McClean (2000) [38] offered a comprehensive review of how population heterogeneity can be incorporated into manpower models. Bartholomew et al. (1991) emphasized that modelling job wastage requires acknowledging the wide range of personal and environmental factors that influence an individual's likelihood of leaving a position. Earlier, Silcock (1954) [35] compiled key determinants affecting job separation tendencies. Additional important contributions include studies by Mehlmann (1979) [26], Bartholomew (1982) [1], and McClean (1978, 1980, 1986, 1993) [21, 22, 23, 24], who illustrated how introducing hypothetical grades can simplify certain semi-Markov models and reduce them to more tractable Markov formulations.

Tsantas (1995) [36] presented a technique for evaluating expectations and central quadratic moments in non-homogeneous Markov systems observed at discrete time points. Later, Tsantas (2001) [37] explored the long-term behaviour of a time-varying Markov model operating within a stochastic environment, highlighting its relevance to manpower analysis. Building on this line of work, Dimitriou and Tsantas (2009, 2010) [4] investigated recruitment strategies in hierarchical structures with multiple entry channels and subsequently generalized these ideas to broader time-dependent Markov manpower models. Guerry (2011) [14] extended the literature by formulating manpower models that integrate both observable characteristics and hidden heterogeneity.

Chattopadhyay and Gupta (2003) [2] developed an occupational mobility framework suited to systems that are partly open and partly closed, while Khan and Chattopadhyay (2003) [20] used stochastic tools to study job mobility among university employees. Models addressing recruitment and promotion mechanisms in airline organizations were proposed by Mukherjee and Chattopadhyay (1989) [30], Chattopadhyay and Gupta (2007) [3], and Gupta and Ghosal (2013) [15]. More recently, Dimitriou and Tsantas (2012) [6] introduced a continuous-time semi-Markov manpower model that incorporated behavioural factors such as seminar participation, along with organizational requirements aimed at preventing shortages of skilled personnel.

Accurately forecasting the number of employees across different hierarchical levels at a specific point in time is essential for effective manpower management. Despite its importance, this issue has not been comprehensively addressed in earlier semi-Markov frameworks. The present study introduces a semi-Markov model designed to predict the distribution of employees across occupational grades at any given moment. Because employees may move from one grade to another at uncertain and uneven intervals, explicitly modelling the holding time in each grade is crucial. The proposed model incorporates these random durations and provides expected workforce numbers across levels within a hierarchical system in which promotions are strictly seniority-based. As a result, it offers a more realistic portrayal of an organization's occupational structure.

Understanding the interval transition probability of an employee is vital, as it determines the eventual destination grade that an employee may reach. This probability describes the likelihood that an employee occupying a given grade at time t had originally entered the system at a different initial grade. An employee recruited at a certain grade may receive a sequence of promotions before retirement, and the timing of these promotions affects the probability of reaching particular destination grades.

The destination probability describes the likelihood of an employee reaching a particular grade before retirement, taking into account the number of promotions required and the holding times associated with each grade. Employees receive retirement benefits based on the grade they hold at the time of retirement, making the determination of destination grades and probabilities highly relevant to organizational planning. In this work, we attempt to compute destination probabilities for employees based on the proposed semi-Markov model.

2. Model

Consider an organization that is arranged into k hierarchical grades. External recruitment is permitted only in the first r grades, where new employees may enter the system alongside the promotion of existing members. For the remaining $(k - r)$ grades, no outside recruitment is allowed, and vacancies are filled exclusively through internal promotions. Hence, grades 1 to r form the open part of the structure, while grades $(r + 1)$ to k constitute the closed segment—an arrangement commonly observed in many manpower systems.

Let $n_i(t)$ denote the number of individuals in grade i at time t, for $i = 1, 2, \ldots, k$. Let $e(t)$ be the number of newly recruited employees at time t. Define p_{ij} as the probability that an employee whose most recent grade was i moves to grade j at the next transition. These probabilities satisfy

$$p_{ij} \geq 0 \quad \text{and} \quad \sum_{j=1}^{k} p_{ij} = 1, \qquad i = 1, 2, \ldots, k.$$

Let $\phi_{ij}(t)$ denote the probability that an individual who enters grade i at time 0 is in grade j at time t. This quantity is referred to as the interval transition probability over the interval $(0, t)$. Let $\tau_{ij}(t)$ be the holding time in grade i prior to moving to grade j, with distribution $h_{ij}(\cdot)$. The mean waiting time $\tilde{\tau}_i$ is related to the mean holding times $\tilde{\tau}_{ij}$ through

$$\tilde{\tau}_i = \sum_{j=1}^{k} p_{ij}\, \tilde{\tau}_{ij}, \qquad i = 1, 2, \ldots, k. \tag{2. 1}$$

In the open grades, 1 through r, the distribution of new entrants is determined by the allocation probabilities p_{0j}, where p_{0j} gives the expected proportion assigned to grade j for $j = 0, 1, 2, \ldots, r$, and $p_{0j} = 0$ for $j = r + 1, \ldots, k$. Thus, the expected number of employees in grade j at time t is

$$E(n_j(t)) = \sum_{i=1}^{k} \phi_{ij}(t)\, E(n_i(0)) + e(t)\, p_{0j}, \qquad j = 1, 2, \ldots, k. \tag{2. 2}$$

Following the approach of Howard (19), we now derive the expression for the interval transition probabilities. Specifically, we seek the probability that an employee in grade i at time 0 will be in grade j at time t. An individual beginning in grade i may receive a first promotion to some intermediate grade q at time m, where $0 < m \leq t$. Further promotions may occur in the remaining period $(t-m)$, eventually leading the individual to grade j at time t. Thus, any transition from grade i to grade j within $(0, t)$ requires at least one intermediate promotion.

Let m be the time at which the first promotion from grade i to grade q occurs. Subsequent transitions then take place in the interval $(t - m)$, resulting in the final promotion to grade j at time t. Under this structure, the interval transition probability $\phi_{ij}(t)$ can be written as

$$\phi_{ij}(t) = \sum_{q=1}^{k} p_{iq} \sum_{m=0}^{t} h_{iq}(m)\, \phi_{qj}(t-m), \qquad i = 1, 2, \ldots, k,\ j = 1, 2, \ldots, k. \tag{2. 3}$$

Equation (2. 3) summarizes the probability of all possible pathways by which an employee starting in grade i may reach grade j by time t. It accounts for every possible intermediate grade q to which the first promotion may occur, and for every possible time m at which this first promotion might take place. The interval transition probabilities are therefore essential for understanding workforce dynamics, since the duration an employee spends in a grade often influences the likelihood of subsequent promotions.

3. Destination Probability of an Employee

In our present model we have considered that an individual entering grade j at time t given that he entered grade i at time 0 with probability $\phi_{ij}(t)$ and $\phi_{ij}(l/t)$ if l promotions have occured before entering grade j from grade i in the time interval $(0, t)$. However, when we observe the number of promotion at time t, we will observe not only the probability that an employee is in grade j but also the probability that he will promoted to next grade q. In our present model, a promotion will occur after a certain time and then destination grade will be determined according as the promotion occurs.

For example, Let us consider the categories of posts in Indian Universities. There are four categories of posts, that are - Assistant Professor-I, Assistant Professor-II, Assistant Professor-III, Associate Professor and Professor. An individual who is now in the post of Assistant ProfessorIII promoted to Associate Professor after 15 years of his service at the age of 55 years. Then his ultimate destination post will be Professor if he will get promotion within 3 years of his last promotion. Otherwise he will get retirement as Associate Professor. So the destination grade of an employee at time of retirement will depend on the time at which the last promotion before his retirement be happen.

So, at first, promotion of employees will happen and then destination grade which is conditional to the promotion time will occur. The promotion that will be occured before retirement be depend on the holding time mass function $h_{jq}(.)$ where an employee is in grade j will be in grade q. The waiting time $w_j(m)$ is unconditional to the destination grade. But here $w_j(m)$ would have to be specified for all the grades. So it will give the probability of getting promotion to each grade given the time when an employee is in present grade before getting promotion to next grade. Let p_{jq} be the probability that an employee who is now in grade j at time m will get promotion to grade q. Therefore, $p_{jq}(m)$ is the transition probability conditional to holding time. Thus it is termed as conditional transition probability.

The conditional transition probability $p_{jq}(m)$ is related to unconditional transition probability p_{jq} with the relationship

$$p_{jq}h_{jq}(m) = p_{jq}(m)w_j(m) \tag{3. 4}$$

where $j = 1(1)k, \quad q = 1(1)k, \quad m = 0(1)t$

The equation (3. 4) states that the probability of the joint event of promotion to grade q and promotion at time m can be expressed in terms of either of the conditional probabilities associated with the joint event. The joint probability is either the product of the transition probability and the holding time probability or the product of waiting time probability and the conditional transition probability.

Thus

$$p_{jq}(m) = \frac{p_{jq}h_{jq}(m)}{w_j(m)} \tag{3. 5}$$

or,

$$h_{jq}(m) = \frac{w_j(m)p_{jq}(m)}{p_{jq}} \tag{3. 6}$$

where $j = 1(1)k, \quad q = 1(1)k, \quad m = 0(1)t$

We now define the quantity $\gamma_{ijq}(l/t)$ as the destination probability that an employee who is in grade i both at time 0 and when no promotion has occured is in grade j at time t has gotten l promotion by that time, and will promoted to next grade q.
So according to this model, the destination probability $\gamma_{ijq}(l/t)$ of an employee will be [19]

$$\gamma_{ijq}(l/t) = \sum_{m=0}^{t} [\phi_{ij}(l/t)p_{jq}h_{jq}(t-m)] \tag{3. 7}$$

where $i = 1(1)k, \quad j = 1(1)k, \quad q = 1(1)k, \quad m = 0(1)t, \quad l = 1, 2, 3, ...$

$\phi_{ij}(l/t)$ is the probability of an employee who is in grade i at time 0 when no promotion occurs given that l promotions have occured for the promotion to grade q at time t. It is then multiplied with the transition probability p_{jq} and holding time distribution $h_{jq}(t-m)$. Taking the sum of these product will give the probability of the destination grade of an employee.

In our model, we have considered that out of k grades r grades are there where direct recruitment is allowed and in the remaining grades no new recruitment from outside is allowed. Here semi-Markov process is used and as it is necessary to consider that how many promotion is needed for an employee to reach his destination grade, the probability $\phi_{ij}(l/t)$ is considered. p_{jq} is the unconditional transition probability of an employee who is in grade j will be in grade q and $h_{ij}(t-m)$ is the conditional holding time distribution that is conditional on the promotion occur at time $(t-m)$. As the destination grade is defined only through the occurances of successive promotions, conditional holding time distribution is considered as it gives the actual senario that how long an employee will stay in a particular grade before getting promotion to next grade.

4. Simulation

A simulation is performed to examine how the destination probabilities $\gamma_{ijq}(l/t)$ vary for different values of the exponential waiting-time rate λ.The exponential distribution is used because its constant hazard rate makes promotion independent of past waiting time, allowing simple and realistic modelling of holding times when detailed data are unavailable. The setup assumes four grades, a maximum time horizon of ten units, and a promotion limit of two. A 4×4 transition matrix P is first generated by drawing each diagonal element p_{ii} for grades $1, 2$, and 3 from a Uniform$(0.6, 0.9)$ distribution and assigning the corresponding promotion probability $p_{i,i+1} = 1 - p_{ii}$. All other elements in these rows are taken as zero. Grade 4 is an absorbing state with $p_{44} = 1$. This structure represents a realistic career system in which individuals either remain in the same grade or move to the next higher one.

For each value of λ, the exponential holding-time distribution

$$h(t; \lambda) = \lambda e^{-\lambda t},$$

is computed for $t = 0, 1, \ldots, 10$ and normalized to form a discrete probability mass function representing the time an individual waits before making a transition. The function $\phi_{ij}(l/t)$ is used to reflect the feasibility and likelihood of moving from grade i to grade j within time t using l promotions. When transitions are possible, it decreases with larger grade differences and longer time periods. The destination probability $\gamma_{ijq}(l/t)$ is then calculated by combining $\phi_{ij}(l/t)$, the transition probability P_{jq}, and the waiting-time distribution across all possible waiting durations. This computation is repeated for each λ from 0.5 to 5.0, generating destination probabilities for every valid path $i \to j \to q$.

Table 1 presents the resulting values of $\gamma_{ijq}(2/10)$. Each row corresponds to a specific combination of initial grade, intermediate grade, and final grade. The probabilities for individuals starting at grade 1 are generally low because reaching higher grades requires multiple upward moves within a limited time. For example, $\gamma_{111}(2/10) = 0.0495$ represents the chance of remaining in grade 1, while $\gamma_{134}(2/10) = 0.0197$ reflects the relatively small likelihood of progressing to grade 4. Individuals beginning at grade 2 show comparatively higher probabilities of reaching advanced grades, reflecting the advantage of entering at a higher level. Those starting in grades 3 and 4 exhibit the largest destination probabilities for remaining in or reaching the highest grade. The value $\gamma_{444}(2/10) = 0.0724$ illustrates the strong absorbing behavior at grade 4. Zero entries, such as $\gamma_{144}(2/10) = 0.0000$, occur when transitions violate the promotion limit and therefore cannot take place.

Overall, the table synthesizes how mobility evolves under an exponential holding-time assumption. It highlights the influence of entry grade, feasible promotion paths and the absorbing nature of the highest level. These probabilities provide important insights into long-term career mobility and lay the groundwork for comparing alternative holding-time distributions in subsequent analyses.

5. Conclusion

In this work, we have developed a semi-Markov process-based model to study occupational mobility and to estimate the destination probabilities of employees within an organization. Unlike standard Markov models, the semi-Markov approach accounts

i	j	q	$\gamma_{ijq}(2/10)$
1	1	1	0.0495
1	1	2	0.0274
1	2	2	0.0639
1	3	3	0.0076
1	3	3	0.0470
1	3	4	0.0197
1	4	4	0.0000
2	2	2	0.0688
2	2	3	0.0081
2	3	3	0.0503
2	3	4	0.0211
2	4	4	0.0628
3	3	3	0.0542
3	3	4	0.0227
3	4	4	0.0673
4	4	4	0.0724

Table 1. Destination probabilities $\gamma_{ijq}(2/10)$ for different transitions.

for the random holding times in each grade, which more realistically represents the uncertain timing of promotions and career progression.

We have demonstrated a method to calculate the destination probabilities $\gamma_{ijq}(l/t)$, which represent the likelihood that an employee starting in grade i will reach grade j after l promotions and eventually move to grade q within a given time period. The simulation study using an exponential waiting-time distribution illustrates how these probabilities vary across different initial grades, intermediate grades and final destination grades. The results show that:

- Higher initial grades increase the likelihood of reaching the final destination grade, reflecting the cumulative advantage of seniority.
- Transitions requiring multiple promotions have lower probabilities, emphasizing the role of promotion frequency and waiting times in shaping career outcomes.
- Absorbing states in the highest grade result in high probabilities of remaining in the top grade once reached, consistent with retirement and career termination effects.
- The approach captures the conditional effect of holding time on promotion probabilities, providing a realistic and detailed view of occupational mobility.

The framework developed in this study can be applied to predict the expected number of employees in each grade at any point in time, which is valuable for manpower planning, human resource management, and policy evaluation in organizations. Moreover, by changing the waiting-time distribution (e.g., Gamma or Weibull), the model can accommodate different organizational and occupational scenarios, making it flexible and widely applicable.

Overall, this work provides a robust quantitative tool for understanding long-term career dynamics and for estimating both the destination grade and the likelihood of reaching it for employees within hierarchical systems. Future studies could extend this framework by incorporating heterogeneity in employee characteristics, external recruitment patterns, and multi-stream promotion systems to further enhance the realism of occupational mobility modeling.

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